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Critical temperature of the deconfining phase transition in (2+1)d Georgi-Glashow model

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Abstract: We find the temperature of the phase transition in the (2+1)d Georgi-Glashow model. The critical temperature is shown to depend on the gauge coupling and on the ratio of Higgs and gauge boson masses. In the BPS limit of light Higgs the previous result by Dunne, Kogan, Kovner, and Tekin is reproduced.

Keywords: Field Theories in Lower Dimensions, Confinement, Thermal Field Theory.

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1. Introduction

Confining gauge theories generally become deconfined at high temperatures. The deconfining phase transition in QCD is the subject of intensive experimental and lattice studies. While some general features of this phase transition can be extracted from symmetry arguments [1], the details cannot be studied analytically due to strong coupling effects resulting from the non-perturbative nature of the phase transition. Therefore it is instructive to investigate theories where deconfinement happens in the weak-coupling regime and hence can be studied quantitatively. One example of such theory, the (2+1)-dimensional SU(2) Georgi-Glashow model, is considered in this paper. The model consists of a SU(2) gauge theory coupled to a scalar (Higgs) field in the adjoint representation with a non-zero VEV v. By studying this model one may hope to find certain features of the deconfining phase transitions which this model shares with QCD. At zero temperature the (2+1)-dimensional SU(N) Georgi-Glashow model could be related through dimensional reduction to high temperature SU(N) gauge theory in four dimensions [2, 3].

It has been shown by Polyakov [4] that 't Hooft-Polyakov monopoles [5] (which are instantons in (2+1)d) give rise to the area law for Wilson loops, i.e., confinement. From the symmetry properties of the Polyakov loops [1] it follows that the Georgi-Glashow model must possess a sharp deconfining phase transition at some temperature. The first analytical attempt to find the temperature of the phase transition in the limit of the small gauge coupling $g \ll v$ was made by Agasyan and Zarembo [6]. Following [4] they argued that at low temperatures ($T \ll M_W$) the theory is equivalent to a 2-dimensional gas of monopoles interacting with each other via (almost massless) Coulomb photons. (Remember that the original SU(2) symmetry is broken to U(1).) The phase transition in the monopole gas would occur through the binding of monopoles and anti-monopoles into pairs similar to Berezinsky-Kosterlitz-Thouless (BKT) phase transition [7]. From this picture they found the critical temperature to be $T_c = g^2/(2\pi)$ [6], where g is the dimensionful gauge coupling. A criticism of this treatment was subsequently raised by Dunne, Kogan, Kovner, and Tekin [8]. They argued that the correct description of the low temperature limit of the Georgi-Glashow model should include W bosons interacting with each other and with the monopoles by exchanging photons. The model thus reduced to a two-dimensional gas of point charges (W's) and monopoles. The presence of thermally excited W bosons decreased the critical temperature by a factor of two yielding $T_c = g^2/(4\pi)$ [8]. The authors of [8] also showed that the W's are required for the phase transition to be of the Z_2 universality class, in agreement with symmetry arguments [1].

In this paper we re-examine the calculation of the critical temperature. We shall show that it actually depends on both the gauge coupling g and the ratio between the Higgs mass m_H and the gauge boson mass m_W. Specifically, our result reads

$$T_c = \frac{g^2}{4\pi}\frac{\epsilon+2}{2\epsilon+1}\,, \tag{1.1}$$

where $\epsilon = \epsilon(m_H/m_W)$ is a function of the ratio of Higgs and W masses M_H and M_W defining the monopole action [9, 10]

$$S_0 = \frac{4\pi v}{g}\epsilon\left(\frac{m_H}{m_W}\right). \tag{1.2}$$

When $m_H \ll m_W$, which is the limit where the Bogomolny-Prasad-Sommerfield (BPS) bound [9] on the monopole action is saturated, $\epsilon(0) = 1$ and T_c in eq. (1.1) coincides with the value found in ref. [8]. Outside this limit, our result disagrees with both refs. [8] and [6].

Our derivation of eq. (1.1) is based on what we believe to be the correct interpretation of the renormalization group approach initiated in ref. [8]. To make the paper self-contained, we will provide alternative derivations for many facts already known in refs. [6, 8]. Readers already familiar with refs. [6, 8] can jump directly to section 5, which is the core of our paper where eq. (1.1) is derived.

The paper is organized as follows. In section 2 we formulate the Georgi-Glashow model, mostly in order to introduce notations. In section 3 we rewrite the partition function in term of a 2d Coulomb gas of electric and magnetic charges. In section 4 we show that the Coulomb gas, in its turn, can be recast into a field theory. We give two equivalent representations of this theory in terms of bosonic and fermionic variables. Our bosonic field theory is different from the one used in [8]. Section 5 is devoted to the renormalization group analysis of the field theory where we determine the value of the critical temperature. Comparison with earlier works and a simple physical argument rederiving eq. (1.1) are given in section 6. In the appendix we review the free massless scalar in two dimensions and show how a bosonic theory for the gas of electric and magnetic charges is constructed.

2. The model

In this paper we shall work mostly in euclidean space. The Georgi-Glashow model is a SU(2) gauge theory where the gauge symmetry is partially broken by a triplet scalar,

$$L = \frac{1}{2}(D_\mu \phi^a)^2 + \lambda(|\phi|^2 - v^2)^2 + \frac{1}{4}F_{\mu\nu}^2 \tag{2.1}$$

In the unitary gauge where $\phi^1 = \phi^2 = 0$, the photon field A^3 is perturbatively massless, while $W^\pm = A^1 \pm iA^2$ are massive gauge bosons with the mass $m_W = gv$ and are charged with respect to the photon field. The theory contains two dimensionless parameters: g/v, which is the expansion parameter of the perturbation theory, and $m_H/m_W \sim \sqrt{\lambda}/v$. We shall assume $g/v \ll 1$ and m_H/m_W parametrically of order one, that is $m_W \sim m_H \gg g^2$.

The model contains a classical solution — the 't Hooft-Polyakov "monopole", which is really an instanton in (2+1)d. (We shall use the terms "monopole" and "instanton" interchangeably.) At large distances, in the unitary gauge, the U(1) field of the instanton is

$$F_{\mu\nu}^3 = \frac{e}{4\pi}\epsilon_{\mu\nu\lambda}\frac{x_\lambda}{x^2}\,, \tag{2.2}$$

where e is the magnetic charge of the monopole,

$$e = \frac{4\pi}{g}\,. \tag{2.3}$$

In our case ge is twice larger than the Dirac quantum 2π. The monopole action is given by eq. (1.2) and is large at weak coupling $g/v \ll 1$; correspondingly the density of instantons is exponentially small, $\sim e^{-S_0}$. Due to the contributions of these instantons to the partition function, photon acquires a mass m_γ which is exponentially suppressed, $m_\gamma \sim e^{-S_0/2}$.

In the euclidean formalism, one puts the field theory at a finite temperature T by making the euclidean time direction periodic with the period $\beta = 1/T$. It turns out that the interesting physics of deconfinement occurs at temperature of order g^2, which is much smaller than W and Higgs masses but is much larger than the photon mass m_γ. We shall concentrate on the behavior of the theory at such temperatures.

3. Coulomb gas representation

Our goal is to rewrite the partition function of the Georgi-Glashow model at finite temperature T such that $m_\gamma \ll T \ll m_W \sim m_H$ in terms of an effective two-dimensional theory. Extending ref. [4], our first step is to rewrite the partition function in terms of a two-dimensional Coulomb gas of monopoles and W bosons,

$$Z = \sum_{M,N=0}^{\infty} \frac{1}{M!N!}\sum_{e_i,g_j}\int \prod_{i=1}^{M} d^2x_i \prod_{j=1}^{N} d^2y_j\, e^{-S(\vec{x}_i,e_i;\vec{y}_j,g_j)} \tag{3.1}$$

where $S(\vec{x}_i, e_i; \vec{y}_j, g_j)$ is the effective action of a system of M monopoles located at the (2d) positions $\vec{x}_i$ with magnetic charges e_i, and N W-bosons located at positions $\vec{y}_j$ with electric charges g_j. The sum over e_i, g_j in eq. (3.1) goes over different signs of the W-boson and monopole charges. The effective action is the sum of three contributions,

$$S(\vec{x}_i, e_i; \vec{y}_j, g_j) = S_{\text{mon}}(\vec{x}_i, e_i) + S_W(\vec{y}_j, g_j) + S_{\text{int}}(\vec{x}_i, e_i; \vec{y}_j, g_j) \tag{3.2}$$

where S_{mon} is the part of the action coming solely from the monopoles, S_W is coming solely from the W's, and S_{int} represents the interaction between the monopoles and the W's.

The explicit form of each term in eq. (3.2) can be found from physically intuitive arguments. For S_{mon}, we notice that the monopole gas is effectively two-dimensional because our assumption about the temperature $T \sim g^2 \gg m_\gamma \sim e^{-S_0/2}$ makes the size of the compactified time direction $(1/T)$ much smaller than the average distance between the monopoles ($\sim e^{S_0/2}$). Thus, the monopoles interact via the 2d Coulomb potential [6], and

$$S_{\text{mon}}(\vec{x}_i, e_i) = MS_0 - \frac{e^2 T}{2\pi} \sum_{ij} e_i e_j \ln(T|\vec{x}_i - \vec{x}_j|)\,, \qquad \sum_i e_i = 0\,. \tag{3.3}$$

Here $e_i = \pm 1$ are the signs of the monopole charges, which are measured in the units of e defined in eq. (2.3). The action in eq. (3.3) integrated over all monopoles spatial positions is finite only when the total magnetic charge vanishes; otherwise it is logarithmically divergent. The reason why T appears in the argument of the logarithms in eq. (3.3) is that T^{-1} is the smallest distance where the inter-monopole potential is logarithmic.

Turning to W bosons, we notice that since $T \ll M_W$ the W's are almost static. As they carry U(1) electric charges they also form a Coulomb gas. We can visualize the worldlines of the W bosons as small circles wrapping around the euclidean time direction; at distances large compared to $\beta = 1/T$, the ensemble of such circles is indistinguishable from a gas of point-like objects. The action S_W is simply the exponent in the Boltzmann suppression factor, i.e., E_W/T where E_W is the energy of the W configuration. Therefore for almost static W's

$$S_W(\vec{y}_j, g_j) = N\frac{m_W}{T} - \frac{g^2}{2\pi T} \sum_{ij} g_i g_j \ln(T|\vec{y}_i - \vec{y}_j|)\,, \qquad g_i = \pm 1\,, \qquad \sum_i g_i = 0\,. \tag{3.4}$$

The first term in the right hand side comes from the static energy, and the second term from the interaction between the W's. Although the contribution of W bosons to the free energy is exponentially suppressed by $e^{-m_W/T}$, they can still destroy confinement, since confinement in this model is due to instantons and the density of instantons is also exponentially suppressed ([8], also see below).

The origin of the interaction term S_{int} is less trivial: it is due to the complex phases acquired by the W bosons along their trajectories (which are closed time loops) in the U(1) field created by the monopoles. This purely imaginary contribution to the effective action of the Coulomb gas has been explicitly computed in ref. [6] to be

$$-S_{\text{int}} = 2i \sum_{i=1}^{N} \sum_{j=1}^{M} e_i g_j \theta(\vec{x}_i - \vec{y}_j) \tag{3.5}$$

where $\theta(\vec{x}_i - \vec{y}_j)$ is the angle between the vector connecting the monopole at $\vec{x}_i$ and the W boson at $\vec{y}_j$ and a chosen spatial direction (say, the x_2 axis). The sum in eq. (3.5) does not depend on the choice of the reference direction if the gas is electrically or magnetically neutral, or both. Thus it is always well-defined when S_{mon} and S_W are finite.

Equation (3.5) can be derived in a simple, almost geometrical, way. Let us consider a simple case of one monopole and and a pair of W^+ and W^- bosons. First we "unwind" the time direction, so that we have W bosons move along finite time intervals in the field of an infinite number of monopoles, positioned periodically along the time axis. It is obvious from figure 1 that the phase obtained by a W boson in this case is the same as when it moves along an infinite temporal trajectory in the field of a single monopole.

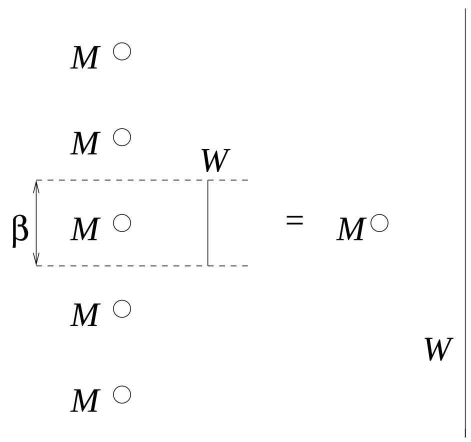

Figure 1: The phase obtained by a W in the field of periodic instantons (see text).

Now the total phase obtained by a pair of W^+ and W^- bosons is proportional to the magnetic flux passing through the two-dimensional surface stretched between the worldlines of the two bosons (see figure 2). If we surround the monopole by a sphere, then the flux is proportional to the area between the two meridians as shown in figure 2. Projected to the equator plane, one sees that flux is indeed proportional to the angle between the line connecting the monopoles to the W^+ and W^- bosons, in agreement with eq. (3.5). The coefficient 2 in eq. (3.5) is due to the fact that the U(1) charge of W bosons is twice the elementary value.

Gathering all terms, we arrive to the Coulomb-gas representation of the partition function of the Georgi-Glashow models at temperatures large compared to m_γ, but small

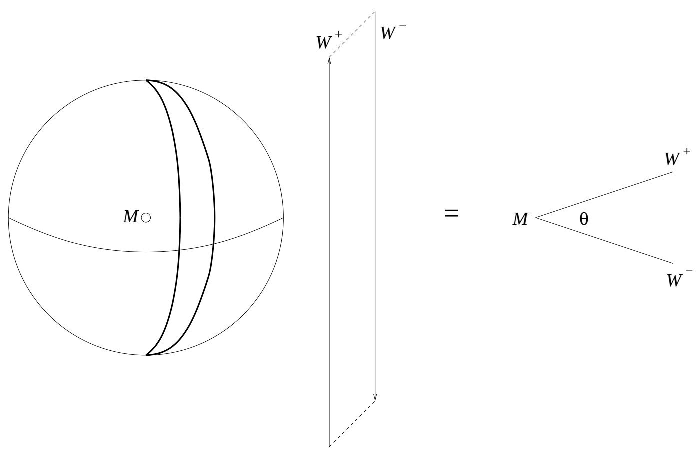

Figure 2: Geometrical interpretation of the phase obtained by a pair of W^+ and W^- bosons in the field of a monopole.

compared to m_W,

$$Z = \sum_{MN} \frac{\zeta_0^M}{M!}\frac{\tilde{\zeta}_0^N}{N!} \sum_{e_i=\pm1}\sum_{g_j=\pm1} \int \prod_{i=1}^M d^2x_i \prod_{j=1}^N d^2y_j \times \tag{3.6}$$
$$\times \exp\left[\frac{e^2T}{2\pi}\sum_{i<j}^M e_ie_j \ln(T|\vec{x}_i-\vec{x}_j|)+\frac{g^2}{2\pi T}\sum_{i<j}^N g_ig_j \ln(T|\vec{y}_i-\vec{y}_j|)+2i\sum_{i=1}^M\sum_{j=1}^N e_ig_j\theta(\vec{x}_i-\vec{y}_j)\right].$$

Here ζ_0 and $\tilde{\zeta}_0$ are the instanton and W fugacities, respectively. They have dimension 2 and are both exponentially suppressed,

$$\zeta_0 \sim T^2 e^{-S_0}\,, \tag{3.7a}$$
$$\tilde{\zeta}_0 \sim T^2 e^{-m_W/T}\,. \tag{3.7b}$$

(The pre-exponents will not be needed for our computation of T_c.) We put the subscript "0" in the notations because these fugacities serve as the initial conditions for the renormalization group in section 5. The sums over e_i and g_i are restricted to configurations with zero total electric and magnetic charges.

4. Effective field theories

In this section we (re)derive effective bosonic and fermionic theories describing a 2d gas of electric and magnetic charges. Our bosonic lagrangian is different from the one obtained by the authors of [8].

4.1 Bosonic theory

By using the result of appendix A, one can write each term in the sum (3.6) as a multipoint Green's function,

$$Z = \sum_{MN} \frac{\zeta_0^M}{M!}\frac{\tilde{\zeta}_0^N}{N!} \sum_{e_i=\pm1}\sum_{g_j=\pm1} \int \prod_{i=1}^M d^2x_i \prod_{j=1}^N d^2y_j \left\langle \mathrm{T}\prod_i e^{ie_i\beta\Phi(\vec{x}_i)}\prod_j e^{ig_j\tilde{\beta}\Theta(\vec{y}_j)}\right\rangle, \tag{4.1}$$

where we have introduced

$$\beta = e\sqrt{T}\,, \qquad \tilde{\beta} = \frac{g}{\sqrt{T}}\,, \tag{4.2}$$

and the average is taken in the free massless bosonic field theory with the ultraviolet cutoff at the scale T. Φ is a (scalar) bosonic field and Θ is the dual field.

One can now perform the summation over e_i and g_j and over M and N in eq. (4.1). The result reads

$$Z = \left\langle T\exp\left[\int d^2x\left(2\zeta_0\cos\beta\Phi + 2\tilde{\zeta}_0\cos\tilde{\beta}\Theta\right)\right]\right\rangle. \tag{4.3}$$

The right hand side, according to the well-known formula of perturbation theory, is equal to

$$Z = \frac{\mathrm{Tr}\, e^{-HT}}{\mathrm{Tr}\, e^{-H_0T}}\,, \tag{4.4}$$

where T is the total euclidean time and

$$H_0 = \int dx \left[\frac{1}{2}(\partial_x \Phi)^2 + \frac{1}{2}(\partial_x \Theta)^2\right], \tag{4.5}$$

$$H = \int dx \left[\frac{1}{2}(\partial_x \Phi)^2 + \frac{1}{2}(\partial_x \Theta)^2 + 2\zeta_0 \cos\beta\Phi + 2\tilde{\zeta}_0 \cos\tilde{\beta}\Theta\right]. \tag{4.6}$$

Thus, our problem is reduced to finding the ground state energy of the hamiltonian H from eq. (4.6), defined with an ultraviolet cutoff at T.

One can construct a (euclidean) lagrangian corresponding to the hamiltonian of eq. (4.6) by requiring that the free part of the lagrangian gives the same correlation functions as in eqs. (A.7) and (A.8). The result is

$$\mathcal{L}_b = \frac{1}{2}(\partial_x \Phi)^2 + \frac{1}{2}(\partial_x \Theta)^2 - i\partial_x \Phi\, \partial_\tau \Theta + 2\zeta_0 \cos\beta\Phi + 2\tilde{\zeta}_0 \cos\tilde{\beta}\Theta\,, \tag{4.7}$$

where Φ and Θ should now be understood as two independent fields. Note that the lagrangian in eq. (4.7) is different from the one derived in [8]. The main difference is that our lagrangian (4.7) is written for two independent scalar fields and the lagrangian used in [8] employs a scalar field χ and its dual field $\tilde{\chi}$ satisfying the duality condition

$$\partial_\mu \tilde{\chi} = -i\epsilon_{\mu\nu}\partial_\nu \chi\,. \tag{4.8}$$

However, the duality condition (4.8) implies $\partial^2\chi = \partial^2\tilde{\chi} = 0$, that is the dual field in the sense of eq. (4.8) exists only for harmonic functions. It is therefore difficult to define Feynman functional integrals over χ for the partition function employing the lagrangian which uses both fields χ and $\tilde{\chi}$ since the integration would have to be restricted to harmonic functions only.

Defining a vector field

$$C_\mu \equiv \partial_\mu \Phi - i\epsilon_{\mu\nu}\partial_\nu \Theta \tag{4.9}$$

the lagrangian of eq. (4.7) could be rewritten in a more Lorentz (rotational)-invariant form

$$\mathcal{L}_b = \frac{1}{2}(\partial_\mu \Theta)^2 + \frac{1}{2}(n_\mu C_\mu)^2 + 2\zeta_0 \cos\beta\Phi + 2\tilde{\zeta}_0 \cos\tilde{\beta}\Theta\,, \tag{4.10}$$

where n_μ is a unit vector in a randomly chosen direction in the (τ, x) plane. Putting $n_\mu = (0, 1)$ one would obtain the lagrangian in eq. (4.7).

In the limit of no monopoles we put $\zeta_0 = 0$ in eq. (4.10). Integrating out the Φ field yields us the lagrangian of the sine-Gordon theory for the 2d gas of W bosons

$$\mathcal{L}_{SG}^{W} = \frac{1}{2}(\partial_\mu \Theta)^2 + 2\tilde{\zeta}_0 \cos\tilde{\beta}\Theta\,. \tag{4.11}$$

Similarly putting $\tilde{\zeta}_0 = 0$ in eq. (4.10) would eliminate W bosons from the theory and would give us a sine-Gordon theory for monopole gas, similar to [6]

$$\mathcal{L}_{SG}^{\mathrm{mon}} = \frac{1}{2}(\partial_\mu \Phi)^2 + 2\zeta_0 \cos\beta\Phi\,. \tag{4.12}$$

The exact solution for the S-matrix of the sine-Gordon theory is known [11] and the exact expression for the soliton mass has been found [12], allowing for quantitative study of the limits shown in eqs. (4.11) and (4.12) along the lines of [6]. An exact solution of the full theory given by eq. (4.10) has not been found. This will prevent us from making quantitative predictions about the behavior of the string tension around T_c but will not interfere with our determination of T_c itself.

Polyakov loops play the role of the order parameter of the deconfining phase transition [1]. To compute the correlation function of two Polyakov loops, one inserts two fundamental charges into the Coulomb gas (3.6). This means that Polyakov loop $P(\vec{x})$ is mapped to the operator $e^{i\tilde{\beta}\Theta/2}$ of the effective theory, so that [6, 8]

$$\langle P(\vec{x})P^\dagger(0)\rangle \sim \left\langle \exp\left(\frac{i}{2}\tilde{\beta}\Theta(\vec{x})\right)\exp\left(-\frac{i}{2}\tilde{\beta}\Theta(0)\right)\right\rangle \tag{4.13}$$

where the average in the right hand side is taken in the theory described by eq. (4.6).

4.2 Fermionic theory

It is easy to apply the bosonization rules [13] to map the bosonic theory (4.6) onto a fermionic theory. We noticed above that without the $\tilde{\zeta}_0 \cos\tilde{\beta}\Theta$ term the lagrangian of eq. (4.10) reduces to a sine-Gordon theory given by eq. (4.12). As was shown in [13] the sine-Gordon theory is equivalent to the massive Thirring model with Dirac mass term. Mandelstam's rules [13] map $\tilde{\zeta}_0 \cos\tilde{\beta}\Theta$ onto a Majorana mass term. Thus, the fermionic effective theory is given by the lagrangian

$$\mathcal{L}_f = \bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{G}{2}(\bar{\psi}\gamma^\mu\psi)^2 + m\bar{\psi}\psi + \frac{\tilde{m}}{2}(\bar{\psi}^c\psi + \bar{\psi}\psi^c) \tag{4.14}$$

where $\psi^c = C\bar{\psi}^T$ is charge-conjugated to ψ. In the chiral basis

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \qquad \psi_c = \begin{pmatrix}\psi_1^\dagger\\ -\psi_2^\dagger\end{pmatrix}. \tag{4.15}$$

The four-fermion coupling G is related to the parameters of the bosonic theory by

$$1 + \frac{G}{\pi} = \frac{4\pi}{\beta^2} = \frac{\tilde{\beta}^2}{4\pi}, \tag{4.16}$$

while the fermion bare masses are

$$m \sim \frac{\zeta_0}{T}, \qquad \tilde{m} \sim \frac{\tilde{\zeta}_0}{T}. \tag{4.17}$$

One can check by an explicit calculation that the lagrangian (4.14) gives the partition function of eq. (3.6).

Defining real Majorana fermion fields [8]

$$P = \frac{\psi + \psi^c}{2}, \qquad Q = \frac{\psi - \psi^c}{2i} \tag{4.18}$$

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we can rewrite the lagrangian of eq. (4.14) as

$$\mathcal{L}_f = \bar{P}\gamma_\mu\partial_\mu P + \bar{Q}\gamma_\mu\partial_\mu Q - 2G\,\bar{P}P\,\bar{Q}Q + (m+\tilde{m})\,\bar{P}P + (m-\tilde{m})\,\bar{Q}Q\,. \qquad (4.19)$$

The physical meaning of the lagrangian in eq. (4.19) will be clarified in the next section.

The bosonic theory (4.7), and, therefore, the partition function, is symmetric under duality transformation $\zeta_0 \leftrightarrow \tilde{\zeta}_0$ and $\beta \leftrightarrow \tilde{\beta}$, or, equivalently under $\Phi \leftrightarrow \Theta$. The duality transformation $\Phi \leftrightarrow \Theta$ corresponds to $\psi_1 \leftrightarrow \psi_1$, $\psi_2 \leftrightarrow \psi_2^\dagger$. This transformation interchanges the Dirac and Majorana mass terms, and the U(1) and U$(1)_A$ currents $j^\mu = \bar{\psi}\gamma^\mu\psi$ and $j^{5\mu} = \bar{\psi}\gamma^\mu\gamma^5\psi$. Naively under this transformation the interaction term changes sign, $G \leftrightarrow -G$, which is not consistent with $\beta \leftrightarrow \tilde{\beta}$ in eq. (4.16). This is because the symmetry between the Dirac and Majorana mass terms in eq. (4.14) is broken by the implicit procedure of regularization: when computing the loop graphs in the theory (4.14) one should use a regularization scheme which preserves the conservation of j^μ when $\tilde{m} = 0$, e.g., the Pauli-Villars scheme with a fermion with large Dirac mass [14]. Using a different regularization scheme, for instance the Pauli-Villars scheme with a heavy Majorana fermion, would result in a relation between fermionic and bosonic couplings different from eq. (4.16) [14].

5. Renormalization group, mass gap and critical temperature

5.1 Review of RG in sine-Gordon theory

At any temperature, the correlation length of the Georgi-Glashow model is equal to the inverse mass gap in the effective theories (4.10) and (4.14). The fact that there are two perturbations on top of the free bosonic theory in eq. (4.10) is crucial for the existence of a critical point with infinite correlation length (zero mass gap). To understand the mechanism of the disappearance of the mass gap, let us recall how the gap emerges when there is only *one* perturbation.

We suppose, for a moment, that $\tilde{\zeta}_0 = 0$. Then our theory is reduced to the sine-Gordon (SG) model of eq. (4.12). The full spectrum of the SG theory is known [11, 12], but for our purposes we only need a method for a parametric estimate of the gap. The Wilson renormalization group (RG) can be employed to this end. In this procedure we decrease the ultraviolet cutoff Λ by successively integrating out the modes with momenta above the cutoff, but below the value of the cutoff at the previous RG step. Since the fugacity ζ_0 will run with Λ we will denote the renormalized fugacity by the function $\zeta(\Lambda)$.

When ζ is small, the perturbation is small and the RG equation for ζ is set solely by the conformal dimension of the operator $\cos\beta\Phi$ [15, 14]

$$\frac{\partial}{\partial\lambda}\frac{\zeta}{\Lambda^2} = (2-\Delta)\frac{\zeta}{\Lambda^2}\,, \qquad \lambda = \ln\frac{T}{\Lambda}\,, \qquad \Delta = \frac{\beta^2}{4\pi}\,, \qquad (5.1)$$

which has an obvious solution

$$\frac{\zeta}{\Lambda^2} = \frac{\zeta_0}{T^2}\left(\frac{T}{\Lambda}\right)^{2-\Delta}\,, \qquad \Delta = \frac{\beta^2}{4\pi}\,. \qquad (5.2)$$

We have written the RG equation at the leading order in the dimensionless coupling ζ/Λ^2 which controls the conformal perturbation theory. If $\beta^2 > 8\pi$ the perturbation is irrelevant since ζ/Λ^2 is smaller than ζ_0/T^2 for all $\Lambda < T$. In the case of $\beta^2 < 8\pi$ where the perturbation is relevant, there exists a scale where $\zeta/\Lambda^2 \sim 1$ and the perturbation is no longer small. Our RG equation (5.1) is not valid beyond this point. The theory becomes strongly coupled at this scale and one can expect the emergence of a mass gap. Thus the mass gap is of the order

$$\Lambda \sim T \left(\frac{\zeta_0}{T^2}\right)^{1/(2-\Delta)} \sim T\, e^{-S_0/(2-\Delta)} \tag{5.3}$$

which agrees parametrically with the result of [12, 11]. The nature of the RG method does not allow the reproduction of the pre-exponents in ref. [12], for which one actually needs the exact expression for the S-matrix of the theory.

5.2 Renormalization group and criterion for T_c

Now we consider our case, where both perturbations are present (see eqs. (4.6) and (4.10)). We now have two RG equations

$$\frac{\partial}{\partial\lambda}\frac{\zeta}{\Lambda^2} = (2-\Delta)\frac{\zeta}{\Lambda^2}\,, \qquad \Delta = \frac{e^2 T}{4\pi} = \frac{4\pi T}{g^2} \tag{5.4a}$$

$$\frac{\partial}{\partial\lambda}\frac{\tilde\zeta}{\Lambda^2} = (2-\tilde\Delta)\frac{\tilde\zeta}{\Lambda^2}\,, \qquad \tilde\Delta = \frac{g^2}{4\pi T}\,. \tag{5.4b}$$

with solutions

$$\frac{\zeta}{\Lambda^2} = \frac{\zeta_0}{T^2}\left(\frac{T}{\Lambda}\right)^{2-\Delta}\,, \qquad \frac{\tilde\zeta}{\Lambda^2} = \frac{\tilde\zeta_0}{T^2}\left(\frac{T}{\Lambda}\right)^{2-\tilde\Delta}\,. \tag{5.5}$$

Noting that $\Delta\tilde\Delta = 1$, one can immediately distinguish three temperature regimes.

(i) $T < g^2/(8\pi)$. In this case $\Delta < 1/2$, $\tilde\Delta > 2$. The operator $\cos\tilde\beta\Theta$ is irrelevant and can be ignored. The infrared properties of the theory is the same as of the sine-Gordon theory (4.12) and hence the theory has a mass gap [6, 12].

(ii) $T > g^2/(2\pi)$. In this case $\Delta > 2$, $\tilde\Delta < 1/2$, the operator $\cos\beta\Phi$ is irrelevant and the theory is in the massive phase of the SG theory of eq. (4.11).

(iii) $g^2/(8\pi) < T < g^2/(2\pi)$. In this case both perturbations are relevant.

The critical point should be in the regime (iii) which we now consider. If we follow the Wilson RG procedure, then at first, when $\Lambda \sim T$, both perturbations are small. As one decreases Λ both perturbations become more and more important. However, if at some scale one perturbation is of order one while the other is still small, then one can conclude that the theory has a mass gap. Indeed, suppose that at some Λ, $\zeta/\Lambda^2 \sim 1$ while $\tilde\zeta/\Lambda^2 \ll 1$. In this case the theory is a SG model with a very small perturbation $\sim \tilde\zeta$. Since the SG is fully gapped, the perturbation theory over $\tilde\zeta/\Lambda^2$ is non-degenerate and does not destroy the existence of the gap. The same happens when $\tilde\zeta/\Lambda^2$ becomes ~ 1 when ζ/Λ^2 is still small. Therefore, the theory can be critical only when ζ/Λ^2 and $\tilde\zeta/\Lambda^2$ are of order one at the same energy scale.

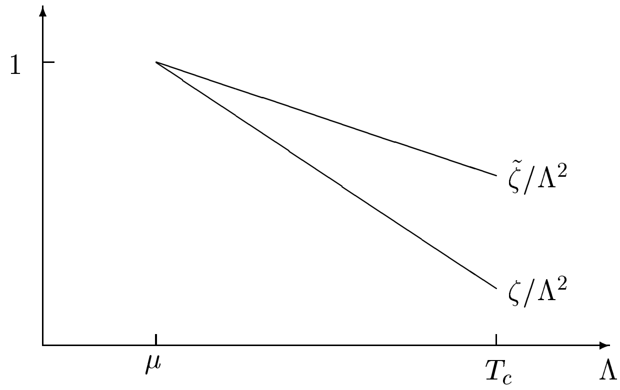

Figure 3: The running of fugacities at $T = T_c$.

In figure 3 we illustrate what happens at T_c. At the scale $T = T_c$ we chose the monopole fugacity ζ to be much smaller than the W fugacity $\tilde{\zeta}$. (In principle it can be the other way around as well.) However, ζ runs faster than $\tilde{\zeta}$, so that at some scale μ the two fugacities become equal and are of the order μ^2. In other words, the perturbative expansion over the two fugacities breaks down at the same scale μ where $\zeta/\mu^2 \sim \tilde{\zeta}/\mu^2 \sim 1$. We can guess that when this condition is satisfied the gap disappears.

If our guess is correct, then T_c can be found be requiring the existence of a scale μ where

$$\frac{\zeta(\mu)}{\mu^2} \sim \frac{\tilde{\zeta}(\mu)}{\mu^2} \sim 1\,. \tag{5.6}$$

Using eq. (3.7) in eq. (5.6) yields

$$e^{-S_0}\left(\frac{T}{\mu}\right)^{2-\beta^2/4\pi} \sim e^{-M_W/T}\left(\frac{T}{\mu}\right)^{2-\tilde{\beta}^2/4\pi} \sim 1\,. \tag{5.7}$$

Solving this equation, taking into account eqs. (1.2) and (4.2), we then find our final result for T_c,

$$T_c = \frac{g^2}{4\pi}\frac{\epsilon+2}{2\epsilon+1}\,,$$

and an estimate for the non-perturbative scale μ at the critical temperature

$$\mu \sim T\exp\left(-\frac{2\epsilon+1}{3}\frac{4\pi v}{g}\right), \qquad T \approx T_c\,. \tag{5.9}$$

We notice, however, that the RG procedure above does not prove that the temperature found above is the critical temperature: it merely says that if the theory becomes critical at some temperature, then this temperature should be given by eq. (1.1).

To argue that the mass gap vanishes at T_c the fermionic lagrangian of eq. (4.19) becomes useful. This can be seen most clearly in the limit of the light Higgs, $m_H \ll m_W$ [8]. (We still have $m_H \gg g^2$, or, equivalently, $m_H/m_W \gg g/v$, so that one loop corrections to the monopole action (1.2) would be finite and eq. (1.2) would still be valid [16].) Then $T_c =$

$g^2/(4\pi)$ with $\beta^2 = \tilde{\beta}^2 = 4\pi$, which leads to $G = 0$ (see eq. (4.16)). The lagrangian (4.19) becomes that of a theory with two free Majorana fermions with masses $m_P = m + \tilde{m}$ and $m_Q = |m - \tilde{m}|$. Using eq. (4.17) with the critical temperature condition eq. (5.6) we may argue that at T_c one may get $m = \tilde{m}$. When $m = \tilde{m}$ we obtain $m_P = 2m$ and $m_Q = 0$. One of the Majorana fermions is massless, and the other one is massive. The existence of the massless Majorana fermion near the phase transition is in agreement with the Ising nature of the phase transition [8]. Notice that, strictly speaking, the RG equations can give us $m_P \sim \mu$, but cannot show that $m_Q = 0$. In the bosonic language, there is no mass gap when $\beta^2 = \tilde{\beta}^2 = 4\pi$ and $\zeta_0 = \tilde{\zeta}_0$ [17].

When the ratio m_H/m_W is large, the fermion coupling G is also large. While we do not know the exact mass spectrum of the theory (4.19), one may guess that qualitatively the picture does not change. This means that near T_c the theory contains two Majorana fermions: a very light one with some mass m_Q, and another one with mass $m_P \sim \mu \gg m_Q$. Because $G \neq 0$ the two fermions interact with each other. However, at energy much below μ, the heavier fermion P decouples, leaving the dynamics to be completely determined by the lighter fermion Q. Since in 2d there is no relevant or marginal self-coupling of one Majorana fermion (e.g., $(\bar{Q}Q)^2 = 0$), the dynamics below μ is determined by one free Majorana fermion, as in the limit $m_H/m_W \to 0$.

5.3 Correlation length, critical region, etc.

It is instructive to find the behavior of the correlator of two Polyakov loops when T is very close to T_c. This correlator has various behaviors at different length scales:

(i) At very large distances $|\vec{x}| \gg M^{-1}$, where M is the mass gap in the theory,

$$\langle P(\vec{x})P(0)\rangle \sim e^{-Mx}\,. \tag{5.10}$$

(ii) In the intermediate range $\mu^{-1} \ll |\vec{x}| \ll M^{-1}$, where μ is defined in eq. (5.9), the correlator behaves that that of the order parameter in the critical Ising model, so that

$$\langle P(\vec{x})P(0)\rangle \sim |\vec{x}|^{-1/4}\,. \tag{5.11}$$

(iii) At distances $T^{-1} \ll |\vec{x}| \ll M^{-1}$, the effective theory is the theory of one massless scalar field, and the behavior of the Polyakov loop correlator is determined by the conformal dimension of the operator $e^{i\tilde{\beta}\Theta/2}$,

$$\langle P(\vec{x})P(0)\rangle \sim |\vec{x}|^{-(2\epsilon+1)/(2\epsilon+4)}. \tag{5.12}$$

It is interesting that this model has Ising-type behavior only at distances larger than μ^{-1}, which is a length scale exponentially large compared to the inverse temperature. At distances between T^{-1} and μ^{-1} the correlator of Polyakov loops has a power-law behavior, but with the critical exponent in eq. (5.12) which does not have an absolute fixed value and can vary continuously depending on m_H/m_W.

From the discussion above it also follows that the critical region, i.e., the range of temperatures where the model exhibits the Ising-like behavior, is very narrow. For a temperature T to be in this region, the fugacities ζ and $\tilde{\zeta}$ should become large ($\sim \Lambda^2$) almost at the same scale, where "almost" means a mismatch by a factor not much larger than one. This gives

$$\frac{|T - T_c|}{T_c} \lesssim \frac{g}{v} \tag{5.13}$$

as the critical region. Deep inside this region, the correlation length exhibits the Ising behavior with the critical index $\nu = 1$:

$$\xi \sim |T - T_c|^{-1}. \tag{5.14}$$

Outside the critical region, the correlation length is set by the energy scale at which one of the perturbations $\cos\beta\Phi$ and $\cos\tilde{\beta}\Theta$ becomes large. Thus,

$$\xi \sim \begin{cases} \dfrac{1}{T}\exp\left(\dfrac{2\pi\epsilon g v}{g^2 - 2\pi T}\right), & T < T_c \\ \dfrac{1}{T}\exp\left(\dfrac{4\pi g v}{8\pi T - g^2}\right), & T > T_c\,. \end{cases} \tag{5.15}$$

One can see from eq. (5.15) that as the temperature increases, the correlation length first increases exponentially from $m_\gamma^{-1} \sim e^{S_0/2}$ at small T to μ^{-1} when T is near the critical region, and then decreases exponentially as T grows past the critical region. Inside the critical region itself, ξ in eq. (5.15) has the same exponential factor as μ^{-1} in eq. (5.9) but the prefactor of the expression for ξ diverges at T_c. Finally we note that in our model, the correlation length of eq. (5.15) is exponentially large for all $T \ll m_W \sim gv$.

6. Discussion and conclusion

We have shown that the phase-transition temperature of the Georgi-Glashow model is given by eq. (1.1). The critical temperature is of order g^2 and also depends on the ratio m_H/m_W via the function $\epsilon(m_H/m_W)$ defining the monopole action through eq. (1.2). In the Georgi-Glashow model ϵ runs from $\epsilon(0) = 1$ to $\epsilon(\infty) \approx 1.787$ [9, 10]. If we formally take $\epsilon \to 0$ limit in eq. (1.1), which of course can never be physically realized in Georgi-Glashow model, we get $T_c = g^2/(2\pi)$. This is the value of the critical temperature suggested in ref. [6]. This is not surprising since when $\epsilon \to 0$ the monopole fugacity becomes $\zeta_0 \sim 1$ while the W bosons' fugacity is still small $\tilde{\zeta}_0 \ll 1$, so that the W bosons are completely outnumbered by the monopoles and the problem reduces to physics of monopole-only gas considered in ref. [6]. In the opposite unphysical limit of $\epsilon \to \infty$, the critical temperature in eq. (1.1) becomes $T_c = g^2/(8\pi)$. In this limit there is no monopoles and the phase transition temperature is simply that of the BKT phase transition for the 2d gas of W's.

In the light Higgs limit $g/v \ll m_H/m_W \ll 1$ corresponding to $\epsilon = 1$ our critical temperature from eq. (1.1) becomes $T_c = g^2/(4\pi)$ in agreement with ref. [8]. For other values of ϵ, or, equivalently, for not extremely small values of the ratio m_H/m_W, our critical temperature is different from the one found in [8].

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The methods used in our paper are almost identical to those of ref. [8]. The crucial difference is that in ref. [8] the RG equation is run to $\lambda = \ln(T/\Lambda) = \infty$, reaching what the authors of [8] interpret to be a fixed point at $\zeta = \tilde{\zeta} = \infty$. In contrast, in our paper the RG equation is used only up to the point where one of the fugacities is of order one. Strictly speaking, the RG equations are derived at the leading order of perturbation theory in ζ/Λ^2 and $\tilde{\zeta}/\Lambda^2$, and thus cannot be used outside the regime $\zeta/\Lambda^2, \tilde{\zeta}/\Lambda^2 \ll 1$.

There exists an intuitive argument leading to our result (1.1). According to this argument, the phase transition occurs when the densities of monopoles and W bosons are equal [8]. Naively the instanton density is e^{-S_0}, but in 2d the instanton action acquires a logarithmic contribution from distances between $1/T$ and the mean inter-instanton distance l_{mon}. Therefore l_{mon} can be obtained from the consistency condition equating the monopole density obtained from geometric considerations $\sim 1/l_{\text{mon}}^2$ to the same density obtained from the modified monopole action. The condition reads

$$\frac{1}{l_{\text{mon}}^2} \sim T^2 \exp\left[-S_0 - \frac{e^2 T}{4\pi} \ln(T l_{\text{mon}})\right]. \tag{6.1}$$

Analogously, the mean distance between the W bosons l_W is given by

$$\frac{1}{l_W^2} \sim T^2 \exp\left[-\frac{m_W}{T} - \frac{g^2}{4\pi T} \ln(T l_W)\right]. \tag{6.2}$$

Solving eqs. (6.1) and (6.2) for l_{mon} and l_W, and requiring $l_m = l_W$, one then obtains eq. (1.1).[1]

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A. Free massless boson in 2d

We consider a free massless boson in two dimensions,

$$S = \int d^2x \, \frac{1}{2}(\partial_\mu \Phi)^2 . \tag{A.1}$$

Although eq. (A.1) is invariant under O(2) rotations, we can choose an arbitrary direction to be the euclidean time axis, and quantize the theory canonically. The canonical commutation relation reads

$$[\Phi(\tau, x),\, \partial_\tau \Phi(\tau, y)] = \delta(x - y)\,, \tag{A.2}$$

[1] Almost the same argument was given in ref. [8], but the logarithmic terms in the exponents were missing.

and the hamiltonian is

$$H = \frac{1}{2}\int dx\,[(-\partial_\tau\Phi)^2 + (\partial_x\Phi)^2]\,. \tag{A.3}$$

We introduce a field Θ defined as follows,

$$\Theta(\tau,x) = i\int_{-\infty}^{x} dy\,\partial_\tau\Phi(\tau,y)\,. \tag{A.4}$$

Since τ is the euclidean time, Θ is a hermitean operator. By definition, $\partial_x\Theta = i\partial_\tau\Phi$. One also finds $\partial_\tau\Theta = -i\partial_x\Phi$ by using the free field equation satisfied by Φ. Thus, $\partial_\mu\Theta = -i\epsilon_{\mu\nu}\partial_\nu\Phi$. We will say that Θ is dual to Φ. The commutation relation between Φ and Θ can also be found from the definition of Θ (A.4),

$$[\Phi(\tau,x),\,\Theta(\tau,y)] = i\theta(y-x)\,. \tag{A.5}$$

In other words, Φ and Θ are mutually non-local. The hamiltonian (A.3) can be written in a form symmetric under duality transformation $\Phi \leftrightarrow \Theta$,

$$H = \frac{1}{2}\int dx\;\left[(\partial_x\Phi)^2 + (\partial_x\Theta)^2\right]\,. \tag{A.6}$$

Since (A.1) is a free field theory, the euclidean Green's functions can be easily computed,

$$\langle \mathrm{T}\Phi(\vec{x})\Phi(0)\rangle = \langle \mathrm{T}\Theta(\vec{x})\Theta(0)\rangle = -\frac{1}{2\pi}\ln(m|\vec{x}|) \tag{A.7}$$

$$\langle \mathrm{T}\Phi(\vec{x})\Theta(0)\rangle = \frac{i}{2\pi}\,\mathrm{sgn}(\tau)\arccos\frac{x}{\sqrt{\tau^2+x^2}} = \frac{i}{2\pi}\theta(\vec{x}) \tag{A.8}$$

where $\vec{x} = (\tau,x)$ and m is a small infrared regulator. In eq. (A.8) the value of arccos is taken to be in $(0,\pi)$. The function $\theta(\vec{x})$ is the angle between the vector $\vec{x} = (\tau,x)$ and the x axis, and runs between $-\pi$ to π. The discontinuity of (A.8) at $\tau = 0$, $x < 0$ is due to the euclidean time ordering,

$$\langle \mathrm{T}\Phi(\epsilon,x)\Theta(0,0)\rangle - \langle \mathrm{T}\Phi(-\epsilon,x)\Theta(0,0)\rangle = \langle [\Phi(0,x),\,\Theta(0,0)]\rangle = i\theta(-x)\,, \tag{A.9}$$

with θ here being the θ-function. The presence of a cut along the spatial axis means that the correlator (A.8) is not invariant under O(2) rotations in the euclidean space-time. This is not surprising, since Φ and Θ are not mutually local.

The following correlator can now be easily computed

$$\begin{aligned}\left\langle \mathrm{T}\prod_{i=1}^{M} e^{i\beta_i\Phi(\vec{x}_i)}\prod_{j=1}^{N} e^{i\tilde{\beta}_j\Theta(\vec{y}_j)}\right\rangle = \left(\frac{\Lambda}{m}\right)^{-((\sum_i\beta_i)^2+(\sum_j\tilde{\beta}_j)^2)/4\pi} \times \\ \times\exp\Bigg[\sum_{i<j}^{M}\frac{\beta_i\beta_j}{2\pi}\ln(\Lambda|\vec{x}_i-\vec{x}_j|) + \sum_{i<j}^{N}\frac{\tilde{\beta}_i\tilde{\beta}_j}{2\pi}\ln(\Lambda|\vec{y}_i-\vec{y}_j|) - \\ -i\sum_{i,j}^{M,N}\frac{\beta_i\tilde{\beta}_j}{2\pi}\theta(\vec{x}_i-\vec{y}_j)\Bigg]\end{aligned} \tag{A.10}$$

where Λ is the ultraviolet cutoff. The m dependence tells us that the correlation function vanishes in the limit $m \to 0$ unless $\sum_i \beta_i = \sum_j \tilde{\beta}_j = 0$ (charge neutrality). The dependence of the correlator on the ultraviolet cutoff Λ is $\Lambda^{-\Delta}$, where

$$\Delta = \sum_i \Delta_i + \sum_j \tilde{\Delta}_j \equiv \sum_i \frac{\beta_i^2}{4\pi} + \sum_j \frac{\tilde{\beta}_j^2}{4\pi} \tag{A.11}$$

is the sum of the conformal dimensions of the operators in the correlator (A.10). The correlator (A.10) does not have branch cuts if all products $\beta_i\tilde{\beta}_j$ are multiple of 2π; in this case, the result does not depend on the choice of the time axis. This is due to the fact that the operators in the correlator (A.10) are mutually local. For the correlator in eq. (4.1) $\beta\tilde{\beta} = 4\pi$ and the partition function is indeed independent of the choice of the time axis.

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